Dimension two vacuum condensates in gauge-invariant theories

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Abstract

Gauge dependence of the dimension two condensate in Abelian and non-Abelian Yang-Mills theory is investigated.

1 Introduction

Recently much attention has been drawn to vacuum condensates $< 0|A_\mu^{a2}|0 >$ and $< 0|\overline{c}^a c^a|0 >$ in non-Abelian gauge theories. It is believed that these condensates carry information about nonperturbative phenomena in quantum chromodynamics, such as quark confinement [1, 2]. They contribute to the nonperturbative parts of the gluon [3, 4] and the quark [5] propagators. In the papers [1, 2] it was suggested that the gluon condensate may be sensitive to various topological defects such as Dirac strings and monopoles. Considered condensates are vacuum expectation values of gauge dependent operators, which makes problems for calculation of observable effects. In the papers [6, 7] it was shown, that if one considers Yang-Mills theory as a limit of a (regularized) noncommutative gauge-invariant theory, then the v.e.v. $< \int d^4xA_\mu^2 >$ doesn't depend on the choice of gauge and, therefore, it may have a direct physical meaning. This proof essentially depends on the existence of a gauge-invariant regularization of noncommutative theories, and this question requires further investigation. Thus, it is interesting to explore the gauge invariance of dimension 2 condensate in the commutative theory, and to study the question of its possible contribution to the Wilson OPE. A partial answer to this question for the case of Abelian theory was given in the work [6]. In this work we continue investigating this question in the Abelian, as well as in non-Abelian cases, and we explore the question of Wilson OPE in noncommutative theory.

2 Some condensates of mass dimension two and their applications in field theory

In this section we will be dealing with Green's functions and v.e.v.'s of the gluon field $A_\mu(x)$ in α-gauges, and of the ghost fields $\overline{c}(x)$, $c(x)$.

The simplest Green's functions in non-Abelian gauge theories are the functions $< TA_\mu(x)A_\nu(y) >$, $< T\overline{c}(x)c(y) >$. Numerical calculations of path integrals, determining these v.e.v.'s, allow to explore the nonperturbative contributions to these propagators. These contributions play an important role in the Wilson expansions of operators $TA_\mu(x)A_\nu(y)$ and $T\overline{c}(x)c(y)$, where they appear as the power corrections to the leading term of order $O((x-y)^{-2})$, which corresponds to the unit operator in the expansion. The next operators, which contribute to this expansion are the operators of mass dimension 2: $A_\mu(x)A^\mu(x)$ and $\overline{c}(x)c(x)$. In this work we will be dealing with these condensates. Their contribution to the Wilson OPE has the following form [8]:

$$\int d^4x e^{ipx}(TA^a_\mu(x)A^b_\nu(0)) \underset{p\to\infty}{\to} C^{[1]ab}_{\mu\nu}(p)\cdot 1 + C^{[A^2_\rho]ab}_{\mu\nu}(p)(A^c_\rho)^2 + C^{[\overline{c}c]ab}_{\mu\nu}(p)\overline{c}^d c^d + ... \quad (1)$$

If one is interested in the behavior of the gluon propagator at large momenta, then one should take the v.e.v. of this expression, which obviously depends on the values of the condensates $< 0|(A^c_\rho)^2|0 >$ and $< 0|\overline{c}^d c^d|0 >$. In particular, it is of interest to know if it is possible to construct gauge-invariant or at least BRST-invariant combinations of these condensates (or corresponding operators). The answer to this question was partially given in the work [8]: in gauges with the gauge fixing/ghost term of the form[1]

$$L_{GF+FP} = \frac{\alpha'}{2}B^aB^a - \frac{\alpha'}{2}gt^{abd}c^b\overline{c}^dB^a + B^a\partial_\mu A^a_\mu + \overline{c}^a M_{ab}c^b - \frac{\alpha'}{8}g^2t^{abd}t^{aef}\overline{c}^b\overline{c}^d c^e c^f \quad (2)$$

there exists a BRST-invariant operator $\mathfrak{O} \equiv \int d^4x(\frac{1}{2}A_\mu(x)A^\mu(x) - \alpha'\overline{c}(x)c(x))$. It is worth noting that BRST-invariance of this operator is preserved in the U(1)-theory in Lorentz-type gauges, but in the general case of Yang-Mills theory in the widely used Lorentz-type gauge (the so-called α-gauge) this statement is no longer correct. No value of α' can turn L_{GF+FP} into

$$L_{GF+FP} = -\frac{\alpha}{2}B^aB^a + B^a\partial_\mu A^a_\mu + \overline{c}^a M_{ab}c^b,$$

which corresponds to Lorentz-type gauge fixing. Besides, one can directly check, that

$$\delta\mathfrak{O} = \int d^4x \left[\partial_\mu(A^a_\mu c_a) + \frac{\alpha}{2}t^{abd}c^b c^d\overline{c}^a\right] = \int d^4x \frac{\alpha}{2}t^{abd}c^b c^d\overline{c}^a \neq 0. \quad (3)$$

On the other hand, in the physical sector the operator $\mathcal{A} \equiv c^b c^d \overline{c}^a$ is equivalent to the null-operator because of the non-zero ghost number:

$$|\psi_{phys}>: \ Q_{ghost}|\psi_{phys}>= Q_{BRST}|\psi_{phys}>= 0$$

[1]Here B is an auxiliary Nakanishi-Lautrup field, and integration over it can be easily done in the path integral

$$0 =< \psi^1_{phys}|[iQ_{ghost}, \mathcal{A}]|\psi^2_{phys} >=< \psi^1_{phys}|\mathcal{A}|\psi^2_{phys} > \tag{4}$$

Note that due to BRST invariance of the above mentioned operator in gauges, determined by the functional (2), and also in Abelian theory, it is easy to see, that BRST-invariance is transferred to the case of maximal Abelian gauge, which has an Abelian sector analogous to the U(1)-theory (more precisely, the $U(1)^{N-1}$-subgroup of the SU(N) group), and in the non-Abelian sector the gauge is fixed by a functional of type (2). It has been also proved in [8].

Let's mention one important difference between the propagators of gauge fields in the Abelian and non-Abelian theories. In Abelian U(1)-theory the gauge parameter α enters the full propagator of the photon field only through the trivial longitudinal part:

$$G_{\mu\nu}(p) = (\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})G(p^2) - \alpha\frac{p_\mu p_\nu}{p^4}, \tag{5}$$

where $G(p^2)$ doesn't depend on α (in non-Abelian theory this statement is false, which is easy to show, making calculations in lowest orders of perturbation theory). The proof of this statement can be found, for instance, in [6]. In the present article a proof, based on the Ward identities, is given in Appendix 1. This observation allows us to build the following gauge-invariant quantity:

$$< 0|T(A_\mu(x)A^\mu(y) + \alpha\overline{c}(x)c(y))|0 > \tag{6}$$

In Abelian theory the ghost field is free, and

$$< 0|T(\overline{c}(x)c(y))|0 >= \int d^4p\ e^{ip(x-y)}\frac{1}{p^2 + i\epsilon}.$$

Taking into account that

$$< 0|T(A_\mu(x)A^\mu(y))|0 >= \int d^4p\ e^{ip(x-y)}G^\mu_\mu(p),$$

we prove gauge invariance of the quantity (6). From analogous considerations it is clear that the vacuum condensate $< 0|A_\mu(x)A^\mu(x) + \alpha\overline{c}(x)c(x)|0 >$ is gauge-invariant, too.

3 Contribution of condensates $< A^2_\mu >$ and $< \overline{c}c >$ to the Wilson expansion of the operator $\mathcal{K}(x,y) \equiv T(A_\mu(x)A^\mu(y) + \alpha\overline{c}(x)c(y))$ in U(1)-theory

In this section we discuss the question, how gauge invariance of the vacuum expectation value $< 0|\mathcal{K}(x,y)|0 >$ constrains the possible terms, contributing to the Wilson expansion of this operator. The Wilson expansion of the operator $\mathcal{K}(x,y)$

in commutative $U(1)$-theory (including interaction with a spinor field $\psi(x)$ of mass m) takes the following form[2]:

$$\mathcal{K}(x,y) \underset{x-y\to 0}{\to} C_0(x-y)\cdot 1 + C^{(1)}_{1\mu\nu}(x-y)A^\mu(y)A^\nu(y) + \\ + C^{(1)}_{2ab}(x-y)\overline{c}^a(y)c^b(y) + ... \tag{7}$$

Coefficients $C^{(1)}_{1\mu\nu}$ and $C^{(1)}_{2ab}$ are dimensionless. Besides, we have just two possible tensor structures: $\eta_{\mu\nu}$ and $z_\mu \equiv x_\mu - y_\mu$. There's one obvious mass scale in the theory - the spinor mass m, and also a hidden one - the subtraction point μ. However, the latter enters only logarithmic corrections to the power expansion (we're considering the case $z \to 0$, but for this expansion to be valid it is necessary that $|g \ln z^2\mu^2| < 1$). Thus, any coefficient function $C(z)$ of the expansion has the following structure:

$$C(z) = Az^\alpha(1 + \sum_{k=1}^{\infty} A_k(g \ln z^2\mu^2)^k) \tag{8}$$

The question of spinor mass is a bit more subtle. Can the terms with negative powers of m contribute to the coefficient functions? If yes, then the whole structure of the Wilson expansion for nonsingular terms is destroyed, because then it would be possible to take operators of arbitrary dimension and to obtain operators of other mass dimension by simply dividing them with the necessary power of mass. As a result in any order of z we would get an infinite series of condensates. Fortunately, here we can apply Weinberg's theorem, which guarantees the existence of a limit of zero spinor mass for quantum electrodynamics in case of diagrams without explicit external momenta, and, therefore, the terms mentioned above are prohibited.

Taking into account all what was said above, let's write down the most general form of the coefficient functions $C^{(1)}_{1\mu\nu}$ и $C^{(1)}_{2ab}$:

$$C^{(1)}_{1\mu\nu} = \beta\eta_{\mu\nu};\ C^{(1)}_{2ab} = \kappa\delta_{ab}; \tag{9}$$

Substituting this in the expansion (7), we obtain:

$$\mathcal{K}(x,y) \underset{x-y\to 0}{\to} C_0(x-y)\cdot 1 + \beta A^2(y) + \kappa\overline{c}^a(y)c^a(y) + ... \tag{10}$$

Gauge invariance of the v.e.v. of the l.h.s. of this equation constrains the coefficients β and κ:

$$\frac{\kappa}{\beta} = \alpha \tag{11}$$

This equality doesn't require any special proof, as we showed in the previous section that the vacuum condensate $< 0|A^2_\mu(x) + \alpha\overline{c}(x)c(x)|0 >$ is gauge independent in the Abelian case.

[2] We don't take into account possible condensates with non-zero ghost number, because they don't contribute to the physical sector of the theory.

4 A necessary consequence of $\frac{d}{d\alpha} < A_\mu^2 >= 0$

The non-Abelian case is especially interesting, though. In the work [6] with the help of noncommutative field theory methods it was shown, that the condensate $< 0|A_\mu^a(x)A_a^\mu(x)|0 >$ is gauge-invariant, but this statement requires some extra explanation, because, for example, from (5) it is clear, that in the particular case of Abelian theory

$$\frac{d}{d\alpha} < 0|A_\mu(x)A^\mu(x)|0 >= D(0),$$

where

$$D(x) = -\int \frac{e^{ipx}}{p^2 + i\epsilon} d^4p,$$

and this quantity is equal to zero only if $D(0) = 0$, which is true, for instance, in dimensional regularization. It is quite probable, that in noncommutative theory the requirement of the existence of a gauge-invariant regularization is rather constraining, and the condition mentioned above is a necessary condition for it. This question is non-trivial and requires further investigation. Here we present one of the possible methods for checking the equality

$$\frac{d}{d\alpha} < A_\mu^2 >= 0, \tag{12}$$

and more precisely we will show that there's a necessary condition:

$$< \overline{c}^a(x)c^a(x) > |_{\alpha=0} = 0, \tag{13}$$

i.e. the absence of ghost condensation in the Lorentz gauge.

The path integral, which determines the vacuum condensate, has the following form in Yang-Mills theory:

$$< \int d^4x A_\mu(x)A^\mu(x) >= \mathrm{N}^{-1} \int \left(\int d^4x A_\mu(x)A^\mu(x) \right) \exp\{i \int d^4x (\mathrm{L}(A_\mu) + \tag{14}$$
$$+ \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \overline{c}^a M_{ab} c^b)\}(\prod_{x,\mu} dA_\mu(x) d\overline{c} dc),$$

where L is the Lagrangian for Yang-Mills fields without matter fields:

$$\mathrm{L} = -\frac{1}{4}\mathrm{tr}(F^{\mu\nu}F_{\mu\nu}) \tag{15}$$
$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + g[A_\mu, A_\nu] \tag{16}$$

Let's make the transformation

$$A_\mu \to A_\mu - \frac{\delta\alpha}{2\alpha} D_\mu^x \int M^{-1}(x,y)\partial^\nu A_\nu(y) d^4y$$

in this path integral, and, keeping only the terms of order $\delta\alpha$, we obtain:

$$< \int\limits_{\alpha} d^4x A_\mu(x) A^\mu(x) >=< \int\limits_{\alpha+\delta\alpha} d^4x A_\mu(x) A^\mu(x) > - \tag{17}$$
$$-\frac{\delta\alpha}{\alpha}\int d^4x \int e^{iS} \mathrm{tr}(A_\mu(x) D^x_\mu(M^{-1}\partial_\nu A^\nu)) \prod dA_\mu(x) d\overline{c} dc.$$

Let's take into account that

$$\mathrm{tr}(A_\mu(x) D^x_\mu(M^{-1}\partial_\nu A^\nu)^x) = A^a_\mu(x) D^{ab}_\mu (M^{-1}_{bc}\partial_\nu A^\nu_c)^x = \tag{18}$$
$$= A^a_\mu(x)(\delta^{ab}\partial_\mu - t^{acb}A^c_\mu(x))(M^{-1}_{bc}\partial_\nu A^\nu_c)^x$$

and the antisymmetry of the structure constants t^{abc} of the gauge group:

$$A^a_\mu(x) A^c_\mu(x) t^{abc} = \frac{1}{2}(A^a_\mu(x) A^c_\mu(x) t^{abc} + A^c_\mu(x) A^a_\mu(x) t^{cba}) = 0. \tag{19}$$

Therefore (17) takes the following form after integration by parts in the last term:

$$< \int\limits_{\alpha} d^4x A_\mu(x) A^\mu(x) >=< \int\limits_{\alpha+\delta\alpha} d^4x A_\mu(x) A^\mu(x) > + \tag{20}$$
$$+\frac{\delta\alpha}{\alpha}\int d^4x d^4y \int e^{iS} \partial^\mu A^a_\mu(x) M^{-1}_{ab}(x,y) \partial^\nu A^b_\nu(y) \prod dA_\mu(x) d\overline{c} dc,$$

where $M(x,y)$ is the kernel of the operator M. Thus, we get:

$$\frac{d}{d\alpha} < \int d^4x A^2_\mu(x) >= -\frac{1}{\alpha}\int d^4x d^4y \int \partial_\mu A^a_\mu(x) M^{-1}_{ab}(x,y) \partial_\nu A^b_\nu(y) e^{iS} \prod dA d\overline{c} dc \tag{21}$$

The r.h.s. of the equation (21) can be rewritten in the following form:

$$\frac{d}{d\alpha} < \int d^4x A^2_\mu(x) >= -\frac{1}{\alpha}\int d^4x\, d^4y < \partial_\mu A^{(0),a}_\mu(x) \partial_\nu A^{(0),b}_\nu(y) >< M^{-1}_{ab}(x,y) > - \tag{22}$$
$$-\frac{1}{\alpha}\int d^4x\, d^4x'\, d^4y\, d^4y' \partial_\mu D^{(0),ac}_{\mu\mu'}(x-x')\, \partial_\nu D^{(0),bd}_{\nu\nu'}(y-y') T^{abcd}_{\mu'\nu'}(x',y').$$

Here $A^{(0),a}_\mu$ is the free gauge field, and $D^{(0),ab}_{\mu\mu'}(x-y)$ is its propagator. The function $T^{abcd}_{\mu'\nu'}$ can be written as a perturbation theory series, using formula (21). It is only important, that this function is nonsingular at $\alpha = 0$. At the same time the free Green's function satisfies the equation

$$\partial^x_\mu D^{(0),ab}_{\mu\nu}(x-y) = -\alpha\delta_{ab}\partial^x_\nu \int \frac{e^{ik(x-y)}}{k^2+i\epsilon} d^4k \tag{23}$$

It follows that in the limit $\alpha \to 0$ the second term in the r.h.s. of (22) is equal to zero, and

$$\frac{d}{d\alpha} < \int d^4x\, A^2_\mu(x) > |_{\alpha=0} =< \int d^4x\, M^{-1}_{aa}(x,x,A) > |_{\alpha=0} =< \overline{c}^a(x) c^a(x) > |_{\alpha=0} \tag{24}$$

In this way we obtain the necessary condition for the gauge independence of the condensate $< \int d^4x A_\mu(x) A^\mu(x) >$:

$$< \int dx\, \bar{c}^a(x) c^a(x) > |_{\alpha=0} = 0 \tag{25}$$

In the Abelian case this requirement reduces to the condition $D(0) = 0$, which was discussed above. Let us explain once again, that the equation (25) should hold in commutative theory, if it is obtained as a limit $\xi \to 0$ from a gauge-invariant regularized noncommutative theory. It is worth noting, that gauge invariance of the condensate $< A_\mu^2 >$ in non-Abelian theory doesn't guarantee its appearance in the Wilson expansion of a gauge-invariant product of operators. The fact is that gauge invariance of this condensate was proved by using the noncommutative formulation of the theory at an intermediate stage. At the same time, as it is shown in Appendix 2, in noncommutative theory the Wilson expansion may be violated by terms of order ξ, which doesn't allow us to make the conclusion about the appearance of this operator in the Wilson expansion.

5 Conclusion.

In this work we have obtained the following results: first of all, in $U(1)$-theory we showed the gauge invariance of the v.e.v. $< 0|T(A_\mu(x)A^\mu(y) + \alpha\bar{c}(x)c(y))|0 >$ and the explicit form of the Wilson expansion of this operator was built. We suggested a method for checking the equation $\frac{d}{d\alpha} < A_\mu^{a2} >= 0$ in the non-Abelian theory by studying the properties of the ghost condensate. It is shown, that in noncommutative field theory the Wilson OPE is no longer valid, and therefore gauge invariance of the condensate $< A_\mu^2 >$ doesn't guarantee, that it contributes to the Wilson expansion of the product of gauge-invariant operators in commutative theory.

Appendix 1.

Proof of the statement $G_{\mu\nu}(p) = (\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})G(p^2) - \alpha\frac{p_\mu p_\nu}{p^4}$; $\frac{dG(p^2)}{d\alpha} = 0$

$$\frac{d}{d\alpha} < T(A_\mu(x)A^\mu(0)) >= \frac{d}{d\alpha}\left[\frac{\int A_\mu(x)A^\mu(0)e^{iS(\alpha)}\prod\limits_x dAd\bar{c}dc}{\int e^{iS(\alpha)}\prod\limits_x dAd\bar{c}dc}\right] \tag{26}$$

It is convenient to introduce the following notation:

$$\begin{aligned} \mathrm{Q}(\alpha) &\equiv \int A_\mu(x)A^\mu(0)e^{iS(\alpha)}\prod_x dAd\bar{c}dc; \\ \mathrm{N}(\alpha) &\equiv \int e^{iS(\alpha)}\prod_x dAd\bar{c}dc; \\ \mathrm{C}(x) &\equiv < T(A_\mu(x)A^\mu(0)) >; \end{aligned} \tag{27}$$

Then

$$\frac{d\mathrm{C}}{d\alpha} = \frac{\mathrm{Q}'\,\mathrm{N} - \mathrm{Q}\,\mathrm{N}'}{\mathrm{N}^2} \tag{28}$$

$$\mathrm{Q}' = \int A_\mu(x)A^\mu(0)\left(-\frac{i}{2\alpha^2}\int(\partial_\mu A^\mu)^2 d^4y\right)e^{iS}\prod dAd\bar{c}dc \tag{29}$$

The Ward identity for Abelian theory takes the form:

$$\frac{1}{\alpha}\partial^y_{\nu_1}\left[\frac{1}{i}\frac{\delta \mathrm{Z}}{\delta J_{\nu_1}(y)}\right] = -\int d^4z J_{\nu_1}(z)\partial^z_{\nu_1}D(y-z)\cdot \mathrm{Z}. \tag{30}$$

Applying the operator $\partial^q_{\nu_2}\left[\frac{1}{i}\frac{\delta}{\delta J_{\nu_2}(q)}\right]$ to this equation, and then the operator $\frac{1}{i^2}\frac{\delta}{\delta J_\mu(x)}\frac{\delta}{\delta J_\mu(0)}$, we obtain:

$$\frac{1}{\alpha}\partial^y_{\nu_1}\partial^q_{\nu_2}\left[\frac{1}{i^2}\frac{\delta^2 \mathrm{Z}}{\delta J_{\nu_1}(y)\delta J_{\nu_2}(q)}\right] = -\frac{1}{i}\delta(y-q)\cdot \mathrm{Z} - \frac{1}{i}\int d^4z J_{\nu_1}(z)\partial^z_{\nu_1}D(y-z)\partial^q_{\nu_2}\frac{\delta \mathrm{Z}}{\delta J_{\nu_2}(q)}; \tag{31}$$

$$\frac{1}{\alpha}\partial^y_{\nu_1}\partial^q_{\nu_2}\left[\frac{1}{i^4}\frac{\delta^4 \mathrm{Z}}{\delta J_{\nu_1}(y)\delta J_{\nu_2}(q)\delta J_\mu(x)\delta J_\mu(0)}\right]_{J=0} = \frac{1}{i^3}\delta(y-q)\frac{\delta^2 \mathrm{Z}}{\delta J_\mu(x)\delta J_\mu(0)}|_{J=0} - \tag{32}$$

$$-\frac{1}{i^3}\times\left\{\partial^x_\mu D(y-x)\partial^q_{\nu_2}\frac{\delta^2 \mathrm{Z}}{\delta J_{\nu_2}(q)\delta J_\mu(0)} - \partial^y_\mu D(y)\partial^q_{\nu_2}\frac{\delta^2 \mathrm{Z}}{\delta J_{\nu_2}(q)\delta J_\mu(x)}\right\}_{J=0}$$

Setting $q = y$ and integrating over y, we get:

$$\mathrm{Q}' = -\frac{i}{2\alpha}\frac{1}{i}\left(-(2\pi)^4(\delta(0))^2\mathrm{Q}\ + \mathrm{N}\int d^4y\partial^y_\mu D(y)\partial^y_{\nu_2} < TA_{\nu_2}(y)A_\mu(x) > - \right. \tag{33}$$

$$\left. - \mathrm{N}\int d^4y\partial^x_\mu D(y-x)\partial^y_{\nu_2} < TA_{\nu_2}(y)A_\mu(0) >\right).$$

Here, of course, by $\delta(0)$ we should understand the regularization $\delta(0) = (2\pi)^{-4}\cdot\Omega$ (Ω is the volume of momentum space) and on having carried out the calculations, we take the limit $\Omega\to\infty$. Integrating by parts in the last two integrals and using the Ward identity for the two-point Green's function

$$\partial^x_\mu\partial^y_\nu < T(A^\mu(x)A^\nu(y)) >= -\alpha\delta(x-y)$$

we obtain the result:

$$\mathrm{Q}' = -\frac{\mathrm{N}}{2\alpha}\left(-(2\pi)^4(\delta(0))^2 < T(A_\mu(x)A^\mu(0)) > -2\alpha D(x)\right) \tag{34}$$

Analogously we get

$$\mathrm{N}' = \frac{1}{2\alpha}(2\pi)^4(\delta(0))^2\mathrm{N} \tag{35}$$

Combining the last two equations, we have:

$$\mathrm{Q}'\cdot\mathrm{N} - \mathrm{N}'\cdot\mathrm{Q} = D(x)\mathrm{N}^2, \tag{36}$$

and due to (28):

$$\frac{d\mathrm{C}}{d\alpha} = D(x). \tag{37}$$

As $C(x)$ is the contraction of the identity (5) with respect to the Lorentz indices, it is clear, that $G(p^2)$ doesn't depend on α.

Appendix 2.

Impossibility of building a Wilson OPE in noncommutative field theory.

Let us demonstrate the impossibility of building a Wilson OPE in noncommutative quantum field theory. We consider the simplest example - the expansion for $T\phi(x)\phi(0)$ as $x \to 0$ in the case of $\phi \star \phi \star \phi \star \phi$-theory. It is clear, that in the noncommutative field theory in principle there could be two different expansions: those with commutative or noncommutative condensates, i.e.:

Variant 1

$$T\phi(x)\phi(0) = C_0(x) \cdot 1 + C_1(x) \cdot [\phi(0) \cdot \phi(0)] + ... \tag{38}$$

Variant 2

$$T\phi(x)\phi(0) = C_0(x) \cdot 1 + C_1(x) \cdot [\phi(0) \star \phi(0)] + ... \tag{39}$$

Neither of these variants is realized. Let's consider a matrix element over one-particle states $< p|T\phi(x)\phi(0)|k >$ in the lowest order[3] ($\sim g$):

$$\begin{aligned} \int d^4x e^{iqx} < p|T\phi(x)\phi(0)|k > \sim \frac{g}{3} \frac{1}{q^2+m^2} \frac{1}{(\omega-q)^2+m^2} \times \\ \times \left(\cos\left[(\omega-q)\times q\right]\cos\left[p\times k\right] + \cos\left[(\omega-q)\times p\right]\cos\left[q\times k\right] + \right. \\ \left. + \cos\left[(\omega-q)\times k\right]\cos\left[q\times p\right]\right), \end{aligned} \tag{40}$$

where $\omega = k - p$. As $q \to \infty$ we get the following asymptotic behavior:

$$\begin{aligned} \underset{q\to\infty}{Asymp} \int d^4x e^{iqx} < p|T\phi(x)\phi(0)|k > \sim \frac{g}{3}\frac{1}{q^4} \cdot \left(\cos\left[(\omega-q)\times q\right]\cos\left[p\times k\right] + \right. \\ \left. + \cos\left[(\omega-q)\times p\right]\cos\left[q\times k\right] + \cos\left[(\omega-q)\times k\right]\cos\left[q\times p\right]\right) \equiv A(q,k,p,\omega) \end{aligned} \tag{41}$$

As it is easy to see, this function $A(q, k, p, \omega)$ cannot be decomposed into a product of two functions, depending on the large momentum q and small external momenta p and k, i.e.

$$A(q, k, p, \omega) \neq B(q) \cdot C(k, p), \tag{42}$$

whereas such representation would be necessary for a Wilson expansion, both (38) or (39) (which is clear directly from the form of these expansions).

For comparison let's consider the commutative limit $\xi \to 0$ of the amplitude (40) or the asymptotic formula (41):

$$\lim_{\xi\to 0} A(q, k, p, \omega) \sim \frac{g}{q^4}, \tag{43}$$

which is factorized in the form (42) if we set:

$$B(q) = \frac{g}{q^4}; C(k, p) = 1. \tag{44}$$

[3]Feynman rules for noncommutative ϕ^4-theory can be found, for instance, in [9]

Similar ideas were presented in the work [10].

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